030207 Quiz 3 Polymer Processing

1)

The *Reynold's Equation*,

$$v_x(y) = \frac{1}{2\eta}\frac{\partial P}{\partial x}y(y-H) + V_0\frac{y}{H}$$

is obtained from the *Navier-Stokes Equation* applied to simple shear flow between two parallel plates. Only the x-component of the *Navier-Stokes Equation* was used to obtain $v_x(y)$ in class,

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \eta\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x \quad \textbf{(x-part of NSE)}$$

a) Many of the terms in the Navier-Stokes equation can be set to 0 by simple assumptions. **-List** (with reference to the effected term) the assumptions involved in obtaining the simplified x-component of the Navier-Stokes equation given below.

$$\frac{\partial P}{\partial x} = \left(\frac{\partial \tau_{xy}}{\partial y}\right) = \eta\left(\frac{\partial^2 v_x}{\partial y^2}\right)$$

b) Through integration and application of limits a simple description of the x-velocity is obtained,

$$v_x(y) = \frac{1}{2\eta}\frac{\partial P}{\partial x}y(y-H) + V_0\frac{y}{H}$$

-Sketch the x-velocity profile as a function of y for the two terms in this expression **-as well as** for the entire Reynold's Equation under an applied pressure gradient and a non-zero plate velocity.

2) The Weissenberg number is a dimensionless number that has been used to estimate the number of entanglements in a polymer melt, We=ψ_1/τ_{12}, where ψ_1 is the first normal stress difference, $\psi_1 = (\tau_{11} - \tau_{22})$ and τ_{12} is the shear stress.

a) Use simple constitutive equations for the first normal stress difference and the viscosity to obtain an expression for We in terms of the rate of strain and constitutive constants.

b) Use this expression and the original definition of the Weissenberg number to explain why "We" might be used as a measure of the number of entanglements.

3) Polymer fluids display steady state rheological features as well as transient rheological features. For example, under steady shear flow the fluid displays a viscosity and a first normal stress difference which are both steady state features. Additionally, if the shear stress is removed the fluid will recoil with a recoverable shear compliance, J_e.

a) Sketch on a log-log plot η, Ψ_1 and γ_e versus the rate of strain.

b) Write an expression relating the recoverable shear compliance at low strain rate, J_e, to the recoverable shear strain and to the shear stress.

c) Give an expression that relates η_0, $\Psi_{1,0}$ and $J_{e,0}$.

d) The expression you wrote for c) means that a polymer fluid displays only two rheological signatures, drag or viscous forces and 3D connectivity or elasticity that lead to normal forces. If the fluid were a 3D transient network (entanglements equally form and dissolve with time) explain in your own words why the recoverable strain would be related to η_0 and $\Psi_{1,0}$.

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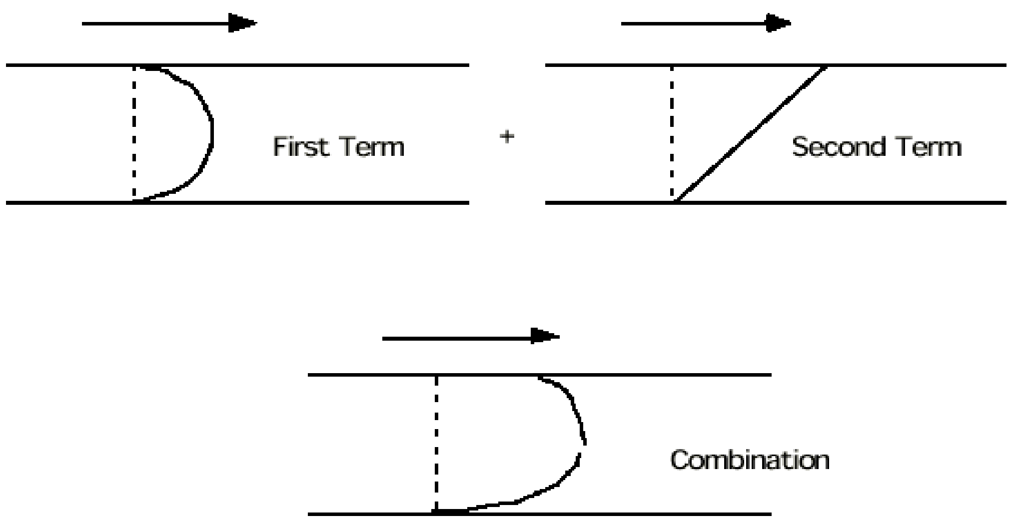
1)

a.) Steady State- First term is 0
Flow only in x direction 3'rd and 4'th term 0
No elongational flow, second term is 0, on right hand side 2'nd term is 0
Infinite plates (no change in x velocity in z direction) 4'th term on RHS is 0
No gravity effects, last term 0.

The equation in terms of shear stress depends on Newton's constitutive equation.

b.)



2)

We = Ψ_1/η $(d\gamma/dt)$. This could be used to estimate entanglements since the presence of entanglements and orientation of chains causes normal stress differences to occur. We = 0 means no entanglements, We = high means many entanglements. The equation indicates that entanglements are associated with high rates of strain and a high ratio between the first normal stress coefficient and the shear viscosity.

The crucial feature is that normal stress results only when there is an entanglement network to transfer shear strain to normal forces. Without entanglements there is no mechanism to develop normal forces. The ratio makes sense since the viscosity reflects in some sense the fraction of shear strain that is transferred to shear stress.

3) a)

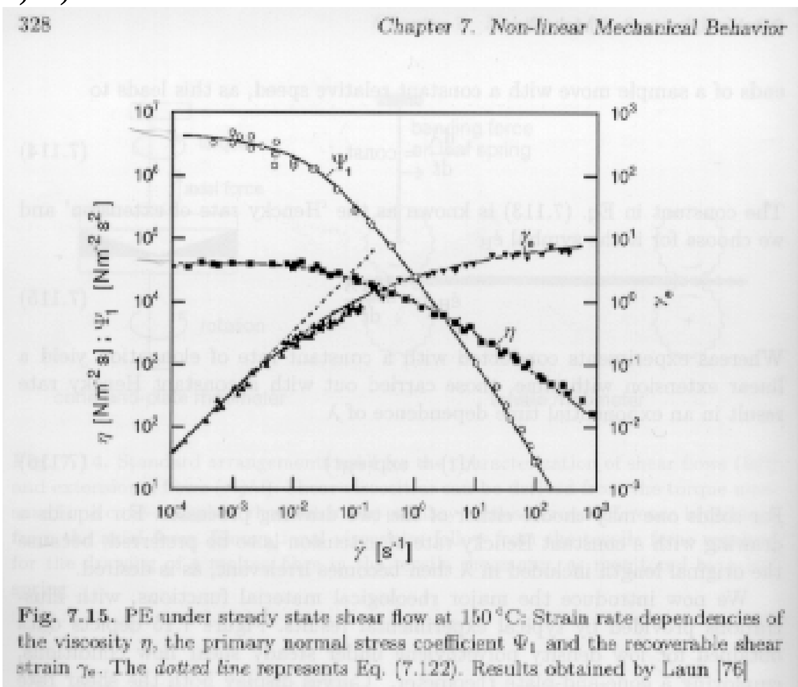


Fig. 7.15. PE under steady state shear flow at 150 °C: Strain rate dependencies of the viscosity η, the primary normal stress coefficient Ψ_1 and the recoverable shear strain γ_e. The *dotted line* represents Eq. (7.122). Results obtained by Laun [76]

b) $J_e(t) = \gamma_e(t)/\tau$

c) $J_{e,0} = \dfrac{\Psi_{1,0}}{2\eta_o^2}$

d) 3D connectivity results in normal forces from shear strain as discussed above. Additionally, the presence of a transient network makes the fluid act as if it is an elastic solid, at least at short times before the memory of fading entanglements disappears. Then there are only two mechanistic concepts that are needed to explain viscosity, normal stresses and recoverable strain, 1) viscous Eyring behavior and 2) a transient network that a) stores elastic energy and b) transfers shear strain to normal forces.